SUMMARY NOTES
ENGI 5557: INTRODUCTION TO ROBOTICS

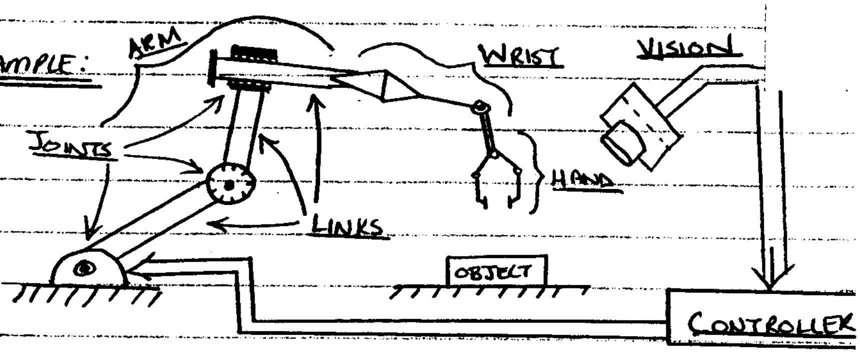
NOTE: IN THIS COURSE ONLY ROBOT MANIPULATORS WILL BE CONSIDERED.

ROBOT MANIPULATOR SYSTEMS (Ch. 1)

COMPOSED OF 3 DISTINCT SUBSYSTEMS.
1. MOTION SUBSYSTEM (JOINTS, LINKS, END-EFFECTOR / HAND)
2. RECOGNITION SUBSYSTEM (VISION / SENSING)
3. CONTROL SUBSYSTEM (CONTROLLER)

ROBOT MANIPULATOR EXAMPLE:

3 PARTS: [ARM, WRIST, HAND]



CLASSIFICATION OF ROBOT MANIPULATORS
(i) POWER SOURCE (ii) APPLICATION AREA (iii) METHOD OF CONTRO

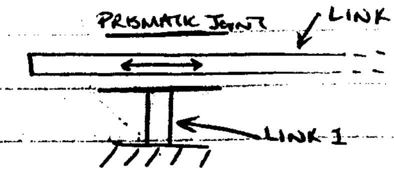
POWER SOURCES
1. ELECTRIC – AC, DC MOTORS BASED ACTUATORS
   – VERY ACCURATE
2. HYDRAULIC – FAST RESPONSE, HIGH TORQUE CAPABILIT
   – NOISY AND REQUIRE HEAVY MAINTENENCE
3. PNEUMATIC – INEXPENSIVE AND SIMPLE
   – NOT VERY PRECISE/ACCURATE

APPLICATION AREAS
1. ASSEMBLY – USUALLY SMALL / ELECTRICALLY DRIVEN
   – " " REVOLUTE OR SCARA IN DESIG
2. NON-ASSEMBLY – WELDING, SPRAY-PAINTING, MATERIAL HANDLING.

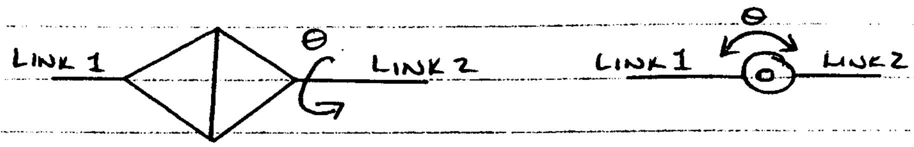
METHOD OF CONTROL
1. NON SERVO – OPEN-LOOP DEVICES
   – LIMITED MOVEMENT
2. SERVO – CLOSED LOOP

JOINTS – TYPICALLY TWO TYPES; (i) PRISMATIC (ii) REVOLUTE

(i) PRISMATIC – CAPABLE OF ONLY TRANSLATION BETWEEN LINKS

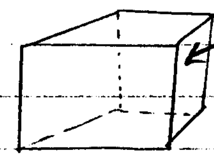


(ii) REVOLUTE – CABABLE OF ONLY ROTATION BETWEEN LINKS



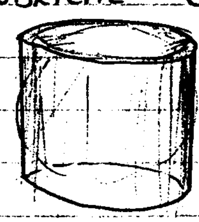
WORKSPACE – REACHABLE SPACE OF A ROBOT MANIPULATOR

(i) CARTESIAN / ORTHOGONAL COORDINATE TYPE

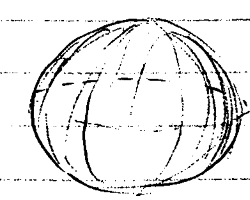


WORKSPACE INCLUDES SOME OR ALL OF A RECTANGLE

(ii) CYLINDRICAL COORDINATE TYPE

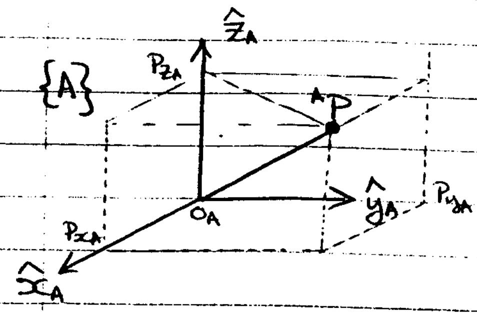


(iii) SPHERICAL COORDINATE TYPE



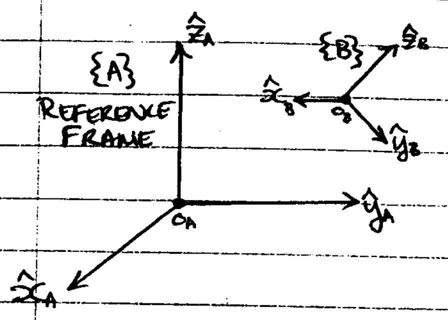
(Ch.2) Rigid Motion and Homogeneous Transformations

Description of a Position:



$${}^AP = \begin{bmatrix} P_{x_A} & P_{y_A} & P_{z_A} \end{bmatrix}^T$$

Description of an Orientation:



Orientation of $\{B\}$ w.r.t. $\{A\}$ is described by:

$${}^A\hat{x}_B = \begin{bmatrix} r_{11} \\ r_{21} \\ r_{31} \end{bmatrix} \quad {}^A\hat{y}_B = \begin{bmatrix} r_{12} \\ r_{22} \\ r_{32} \end{bmatrix} \quad {}^A\hat{z}_B = \begin{bmatrix} r_{13} \\ r_{23} \\ r_{33} \end{bmatrix}$$

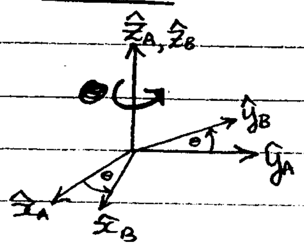
* The components r_{ij} of each vector are the projection of each vector $\hat{x}_B, \hat{y}_B$ and $\hat{z}_B$ onto the unit direction of ref. frame.

$${}^A_BR = \begin{bmatrix} \hat{x}_B\cdot\hat{x}_A & \hat{y}_B\cdot\hat{x}_A & \hat{z}_B\cdot\hat{x}_A \\ \hat{x}_B\cdot\hat{y}_A & \hat{y}_B\cdot\hat{y}_A & \hat{z}_B\cdot\hat{y}_A \\ \hat{x}_B\cdot\hat{z}_A & \hat{y}_B\cdot\hat{z}_A & \hat{z}_B\cdot\hat{z}_A \end{bmatrix}$$

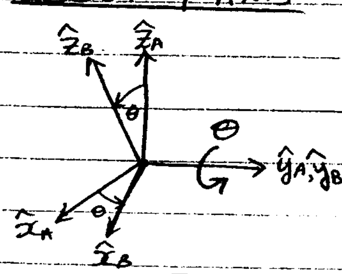
<u>Note</u>: $\hat{y}_B\cdot\hat{y}_A$ gives the projection of $\hat{y}_B$ onto $\hat{y}_A$

Rotation About an Axis

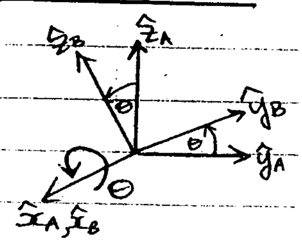
Rotation About Z-axis



About Y-axis



About ~~Z~~ X-axis



$${}^A_BR = R_{z_A}(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad R_{y_A}(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \quad R_{x_A}(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -s \\ 0 & \sin\theta & co \end{bmatrix}$$

④

<u>NOTE:</u> THE THREE PREVIOUS ROTATIONAL MATRICES $R_{ZA}(\theta)$, $R_{YA}(\theta)$ AND $R_{XA}(\theta)$ ARE <u>ORTHOGONAL</u> i.e $\boxed{{}^{A}_{B}R^{-1} = {}^{A}_{B}R^{T}}$

ALSO,

$$ {}^{A}_{B}R = \left[{}^{A}\hat{x}_{B} \;\; {}^{A}\hat{y}_{B} \;\; {}^{A}\hat{z}_{B} \right] = \begin{bmatrix} {}^{B}\hat{x}_{A}^{T} \\ {}^{B}\hat{y}_{A}^{T} \\ {}^{B}\hat{z}_{A}^{T} \end{bmatrix} $$

$$ \therefore \boxed{ {}^{B}_{A}R = {}^{A}_{B}R^{T} = {}^{A}_{B}R^{-1} } $$

<u>PROPERTIES OF ROTATIONAL MATRICES</u>

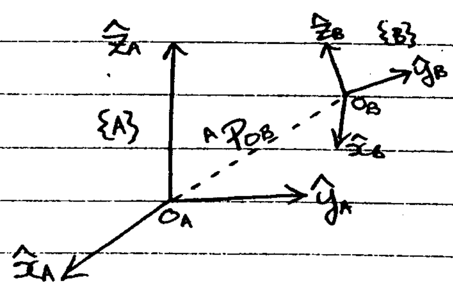
① $R_{ZA}(\emptyset) = I$

② $R_{ZA}(\theta) \cdot R_{ZA}(\phi) = R_{ZA}(\theta + \phi)$

③ $R_{ZA}^{-1}(\theta) = R_{ZA}(-\theta)$

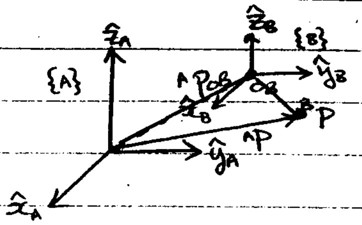
<u>DESCRIPTION OF A FRAME</u>

INFORMATION REQUIRED TO DESCRIBE THE WHERE ABOUTS OF AN ARTBITRARY FRAME IN SPACE W.R.T. A REFERENCE/FIXED FRAME IS AN <u>ORIENTATION</u> AND A <u>POSITION</u>.



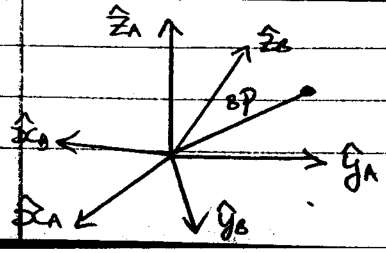
✱ THE FRAME $\{B\}$ IS DESCRIBED BY ${}^{A}P_{OB}$ AND ${}^{A}_{B}R$ (ROTATIONAL MATRIX RELATING $\{B\}$ TO $\{A\}$)

<u>MAPPING PURE TRANSLATION OF A FRAME</u>



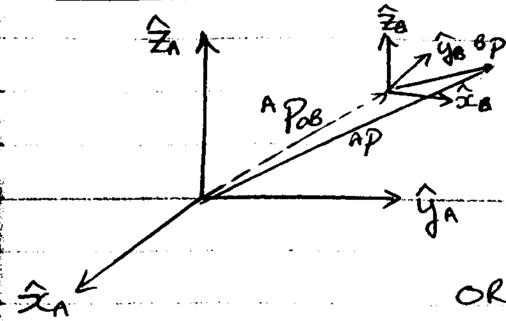
$$ \boxed{ {}^{A}P = {}^{A}P_{OB} + {}^{B}P } $$

<u>MAPPING PURE ROTATION OF A FRAME</u>



$$ \boxed{ {}^{A}P = {}^{A}_{B}R \, {}^{B}P } $$

MAPPING INVOLVING TRANSLATION AND ROTATION



$$\boxed{{}^AP = {}^A_BR\,{}^BP + {}^AP_{OB}}$$

OR, HOMOGENEOUS TRANSFORMATION FORM

$$\begin{bmatrix} {}^AP \\ 1 \end{bmatrix} = \left[\begin{array}{c|c} {}^A_BR & {}^AP_{OB} \\ \hline \bar{0} & 1 \end{array}\right] \begin{bmatrix} {}^BP \\ 1 \end{bmatrix}$$

LET: $\underbrace{{}^A_BT = \left[\begin{array}{c|c} {}^A_BR & {}^AP_{OB} \\ \hline \bar{0} & 1 \end{array}\right]}_{\text{HOMOGENEOUS TRANSFORMATION}}$

$$\boxed{\begin{bmatrix} {}^AP \\ 1 \end{bmatrix} = {}^A_BT \begin{bmatrix} {}^BP \\ 1 \end{bmatrix}}$$

SIMPLER NOTATION: $\boxed{{}^AP = {}^A_BT\,{}^BP}$

NOTE: $\boxed{{}^B_AT = {}^A_BT^{-1} \neq {}^A_BT^T}$

REMARKS: PURE ROTATION: $${}^A_BT = \left[\begin{array}{c|c} {}^A_BR & \begin{matrix}0\\0\\0\end{matrix} \\ \hline \bar{0} & 1 \end{array}\right]$$

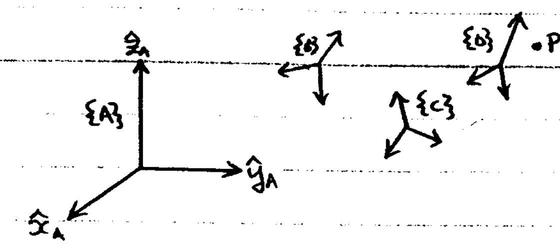
PURE TRANSLATION: $${}^A_BT = \left[\begin{array}{c|c} \begin{matrix}1&0&0\\0&1&0\\0&0&1\end{matrix} & {}^AP_{OB} \\ \hline \bar{0} & 1 \end{array}\right]$$

$${}^A_BT = \left[\begin{array}{c|c} {}^A_BR & {}^AP_{OB} \\ \hline \bar{0} & 1 \end{array}\right] \Longrightarrow {}^B_AT = {}^A_BT^{-1} = \left[\begin{array}{c|c} {}^A_BR^T & -{}^A_BR^T\,{}^AP_{OB} \\ \hline \bar{0} & 1 \end{array}\right]$$

⑥

Homogeneous Transformations: In Cases Involving Multiple Frames and Some Fixed Ref. Frame

i.e



Given ${}^{D}P$, Find ${}^{A}P$

$$\left.\begin{aligned} {}^{C}P &= {}^{C}_{D}T\,{}^{D}P \\ {}^{B}P &= {}^{B}_{C}T\,{}^{C}P \\ {}^{A}P &= {}^{A}_{B}T\,{}^{B}P \end{aligned}\right\} \quad \begin{aligned} &{}^{A}P = {}^{A}_{B}T\,{}^{B}_{C}T\,{}^{C}_{D}T\,{}^{D}P \\ &\boxed{{}^{A}P = {}^{A}_{D}T\,{}^{D}P} \end{aligned}$$

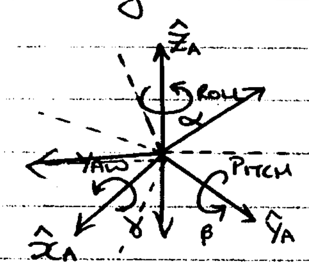
Where:

$${}^{A}_{D}T = {}^{A}_{B}T\,{}^{B}_{C}T\,{}^{C}_{D}T$$

$${}^{A}_{D}T = \left[\begin{array}{c|c} {}^{A}_{B}R\,{}^{B}_{C}R\,{}^{C}_{D}R & {}^{A}_{B}R\,{}^{B}_{C}R\,{}^{C}P_{OD} + {}^{A}_{B}R\,{}^{B}P_{OC} + {}^{A}P_{OB} \\ \hline 0\ 0\ 0 & 1 \end{array}\right]$$

Other Methods to Describe Rotation

X-Y-Z Fixed Angles (Roll, Pitch, Yaw Angles)



* Start with {B} coincident with {A} *

① Rotate {B} about $\hat{X}_A$ by angle γ (Yaw angle)

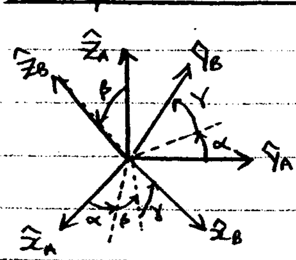
② Rotate {B} about $\hat{Y}_A$ by angle β (Pitch angle)

③ Rotate {B} about $\hat{Z}_A$ by angle α (Roll angle)

$$\boxed{{}^{A}_{B}R = R_Z(\alpha)\cdot R_Y(\beta)\cdot R_X(\gamma)} = {}^{A}_{B}R_{\underbrace{XYZ}_{\text{Fixed}}}(\gamma,\beta,\alpha)$$

Note: 3DOF; and rotation matrices are multiplied in the reverse order.

Z-Y-X Euler Angles (Rotations Relative to Current Axis)



* Start with {B} coincident with {A} *

① Rotate {B} about $\hat{Z}_B$ by angle α

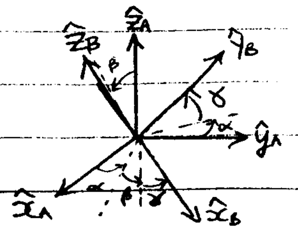
② Rotate {B} about $\hat{Y}_B$ by angle β

③ Rotate {B} about $\hat{X}_B$ by angle γ

Note: 3 DOF; and rotation matrices multiplied in order of rotation

$$\boxed{{}^{A}_{B}R_{\underbrace{ZYX}_{\text{Moving}}} = R_Z(\alpha)\cdot R_Y(\beta)\cdot R_X(\gamma)} = {}^{A}_{B}R_{\underbrace{XYZ}_{\text{Fixed}}}$$

Z - Y - Z EULER ANGLES (ROTATIONS RELATIVE TO CURRENT AXIS)



* START WITH {B} COINCIDENT WITH {A} *

① ROTATE {B} ABOUT $\hat{z}_B$ BY ANGLE α

② ROTATE {B} ABOUT $\hat{y}_B$ BY ANGLE β

③ ROTATE {B} ABOUT $\hat{z}_B$ BY ANGLE γ

$$\boxed{{}^{A}_{B}R_{\underbrace{ZYZ}_{\text{MOVING}}} = R_z(\alpha)\cdot R_y(\beta)\cdot R_z(\gamma)}$$ ← FUNCTION OF 3 VARIA… ∴ 3DOF

FORWARD KINEMATICS (Ch. 3)

(CONVENTIONALLY USES DENAVIT – HARTENBERG REPRESENTATION)

CONVENTIONS/NOTATIONS:

- SUPPOSE ROBOT HAS $(n+1)$ LINKS; $(0 \rightarrow n)$
- BASE OF ROBOT = LINK 0
- JOINTS NUMBERED 1 TO n; ith JOINT = LINKS $(i-1)$ AND i CONNEC…
- COORDINATE FRAME ATTACHED TO EACH LINK
  - ↳ FRAME 0: FIXED/REFERENCE FRAME, ATTACHED TO BASE (LINK …
  - ↳ FRAME i: ATTACHED TO LINK i
- SUPPOSE A_i IS THE HOMOGENEOUS TRANSFORMATION FROM FRAME i TO FRAME $i-1$: $\boxed{A_i = A_i(q_i)}$, AND IS A FUNCTION OF JOINT VARIABLE q_i

$$q_i = \begin{cases} \theta_i & \text{FOR REVOLUTE JOINTS (ANGLE…} \\ d_i & \text{FOR PRISMATIC JOINTS (TRANSLA…} \end{cases}$$

THEN THE HOMOGENEOUS TRANSFORMATION THAT TRANSFORMS A POINT IN FRAME j TO A POINT IN FRAME i IS GIVEN BY:

$$\boxed{{}^{i}_{j}T = A_{i+1}A_{i+2}\cdots\cdots A_{j-1}A_j} \quad (i<j)$$

$$\boxed{{}^{i}_{j}T = I} \quad (i=j)$$

$$\boxed{{}^{i}_{j}T = \left({}^{j}_{i}T\right)^{-1}} \quad (\forall j, i)$$

$$\boxed{A_i = \left[\begin{array}{c|c} {}^{(i-1)}_{\ \ i}R & {}^{(i-1)}_{\ \ i}P \\ \hline 0\,0\,0 & 1 \end{array}\right]} \qquad \boxed{{}^{i}_{j}T = \left[\begin{array}{c|c} {}^{i}_{j}R & {}^{i}_{j}P \\ \hline 0\,0\,0 & 1 \end{array}\right]}$$

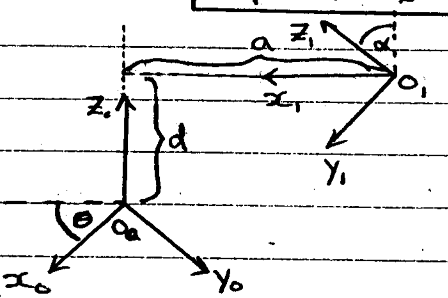
THE DENAVIT - HARTENBERG (D-H) REPRESENTATION

FORWARD KINEMATICS: GIVEN JOINT VARIABLES q_i OF THE ROBOT, DETERMINE THE POSITION AND ORIENTATION OF THE END-EFFECTOR

* CONSIDER FRAME {0} (x_0, y_0, z_0) AND FRAME {1} (x_1, y_1, z_1) *

IF (DH1) $\boxed{x_1 \perp z_0}$ AND (DH2) $\boxed{x_1 \text{ INTERSECTS } z_0}$ THEN THERE EXISTS UNIQUE PARAMETERS a, d, θ, AND α SUCH THAT THE HOMOGENEOUS TRANSFORMATION FROM {1} TO {0} IS DESCRIBED BY:

$$\boxed{A_1 = Rot_z(\theta) \cdot Tran_z(d) \cdot Tran_x(a) \cdot Rot_x(\alpha)}$$



OR IN GENERAL,

$$\boxed{A_i = R_z(\theta_i) \cdot Tran_z(d_i) \cdot Tran_x(a_i) \cdot R_x(\alpha_i)}$$

$$A_i = \left[\begin{array}{ccc|c} C_{\theta_i} & -S_{\theta_i} C_{\alpha_i} & S_{\theta_i} S_{\alpha_i} & a_i C_{\theta_i} \\ S_{\theta_i} & C_{\theta_i} C_{\alpha_i} & -C_{\theta_i} S_{\alpha_i} & a_i S_{\theta_i} \\ 0 & S_{\alpha_i} & C_{\alpha_i} & d_i \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

α - TWIST
a - LENGTH
d - OFFSET
θ - ANGLE

NOTE: ONLY FOUR PARAMETERS ARE REQUIRED TO DESCRIBE A POINT

STEPS IN SETTING UP FRAMES (FOR D-H REPRESENTATION)

STEP
1) LOCATE AND LABEL JOINT AXES $z_0, \ldots, z_{n-1}$
2) ESTABLISH THE BASE FRAME (ORIGIN O_0 ANYWHERE ON z_0, MAKE RIGHT-HANDED FRA
3) LOCATE ORIGIN O_i WHERE COMMON NORMAL ~~TO~~ BETWEEN z_i AND z_{i-1} INTERSECTS z_i
   NOTE: ① IF z_i INTERSECTS z_{i-1}, LOCATE O_i AT THAT POINT
   ② IF z_i IS PARALLEL TO z_{i-1}, LOCATE O_i AT JOINT i.
4) ESTABLISH x_i ALONG COMMON NORMAL BETWEEN z_{i-1} AND z_i THROUGH O_i OR IN THE DIRECTION NORMAL TO $z_{i-1} - z_i$ PLANE IF THEY INTERSECT
5) ESTABLISH y_i TO COMPLETE A RIGHT HAND FRAME.
6) ESTABLISH THE END-EFFECTOR FRAME

SETTING UP A TABLE OF LINK PARAMETERS

LINK	a_i	d_i	α_i	θ_i
1	a_1	d_1	α_1	θ_1
2	a_2	d_2	α_2	θ_2
⋮	⋮	⋮	⋮	⋮

a_i: DISTANCE ALONG x_i FROM O_i TO $x_i - z_{i-1}$ INTERS
d_i: DISTANCE ALONG z_{i-1} FROM O_{i-1} TO $x_i - z_{i-1}$ INTERSEC
α_i: ANGLE BETWEEN z_{i-1} AND z_i ABOUT x_i
θ_i: ANGLE BETWEEN x_{i-1} AND x_i ABOUT z_{i-1}

INVERSE KINEMATICS (Ch 4)

- GIVEN POSITION AND ORIENTATION OF END EFFECTOR DETERMINE THE CORRESPONDING JOINT VARIABLES $q_1, q_2, \ldots, q_n$.

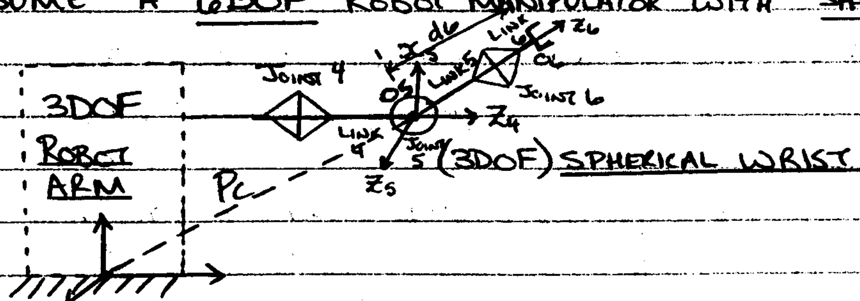
PROBLEMS ENCOUNTERED FOR INVERSE KINEMATICS:

1) POSITION TYPICALLY GIVEN IN TERMS OF A NONLINEAR FUNCTION OF THE JOINT VARIABLES, THEREFORE A 'CLOSED FORM' SOLUTION IS DIFFICULT TO FIND
2) SOLUTIONS MAY NOT EXIST (e.g. OUT OF WORKSPACE)
3) AN INFINITE NUMBER OF SOLUTIONS MAY EXIST (e.g. SINGULAR POSITIONS)
4) CAN HAVE MULTIPLE SOLUTIONS FOR SAME POSITION (e.g. MORE THAN 1 POSSIBLE CONFIGURATION)

KINEMATIC DECOUPLING

- ALGORITHM ~~FOR~~ USEFUL FOR SOLVING THE INVERSE KINEMATICS PROBLEM FOR ROBOT MANIPULATORS WITH END EFFECTOR JOINT AXIS INTERSECTING AT A SINGLE POINT KNOWN AS THE "WRIST CENTER"

ASSUME A 6DOF ROBOT MANIPULATOR WITH 'SPHERICAL WRIST'



KINEMATIC DECOUPLING IS BASED ON THE IMPORTANT FACT THAT THE WRIST HAS A WRIST CENTER, P_c, WHERE ALL JOINT AXIS INTERSECT.

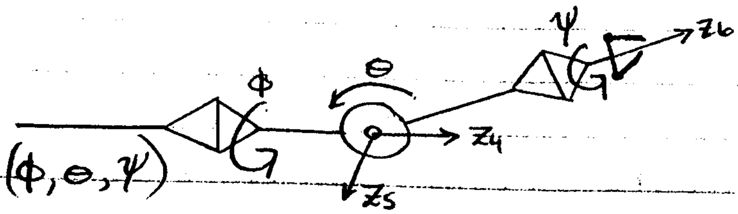
PROCEDURE:

1. OBTAIN DH REPRESENTATION FOR THE ROBOT MANIPULATOR
2. GIVEN DESIRED POSITION $d = \begin{bmatrix} d_x \\ d_y \\ d_z \end{bmatrix}$ AND ORIENTATION $R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$ OF THE END EFFECTOR DETERMINE THE WRIST CENTER POSITION EXPRESSED IN BASE FRAME: $P_c = \begin{bmatrix} P_{cx} \\ P_{cy} \\ P_{cz} \end{bmatrix}$

$$\boxed{P_c = d - R\begin{pmatrix} 0 \\ 0 \\ d_6 \end{pmatrix}}$$

3. USING P_{cx}, P_{cy}, P_{cz} DETERMINE GEOMETRICALLY $q_1, q_2, q_3 \xrightarrow{\text{THEN}} {}^0_3R$
4. DETERMINE $\boxed{{}^3_6R = {}^0_3R^T R}$
5. FIND A SET OF EULER ANGLES $\theta_4, \theta_5, \theta_6$ SATISFYING ${}^3_6R$ FOUND IN 4

SPHERICAL WRIST (EULER ANGLES & SINGULAR CONFIG'S)



EULER ANGLES: (ϕ, θ, ψ)

$$ {}^{3}_{6}R = R_z(\phi)\, R_y(\theta)\, R_z(\psi) = \begin{bmatrix} (C\phi C\theta C\psi - S\phi S\psi) & (-C\phi C\theta S\psi - S\phi C\psi) & C \\ (S\phi C\theta C\psi + C\phi S\psi) & (-S\phi C\theta S\psi + C\phi C\psi) & S \\ -S\theta C\psi & S\theta S\psi & C \end{bmatrix} $$

SINGULARITIES: $\theta = 0, \pi$; CAN BE PROOVED MATHEMATICALLY BY ANALYSIS OF THE ABOVE ROTATION MATRIX FOR EULER ANGLE (ϕ, θ, ψ) OR ANALYTICALLY BY OBSERVATION OF THE PHYSICAL MECHANICS OF THE SPHERICAL WRIST SHOWN ABOVE.

(Ch5) VELOCITY KINEMATICS – THE MANIPULATOR JACOBIAN

- DERIVATIONS COME FROM PROPERTIES OF SKEW SYMMETRIC MATRICES (SEE TEXTBOOK OR FULL NOTES)

LET $ {}^{0}_{n}T(q) = \begin{bmatrix} {}^{0}_{n}R(q) & {}^{0}_{n}d(q) \\ 0 & 1 \end{bmatrix}$ DENOTE THE HOMOGENEOUS TRANSFORMATION FROM END-EFFECTOR FRAME TO BASE FRAME

* n DOF MANIPULATOR $\longrightarrow$ ($q = [q_1\ q_2 \cdots q_n]^T$ ARE THE SET OF JOINT VARIABLES)

FORWARD KINEMATICS

$$ \boxed{\begin{bmatrix} {}^{0}_{n}V \\ {}^{0}_{n}\omega \end{bmatrix} = \begin{bmatrix} J_v \\ \hline J_\omega \end{bmatrix} \dot{q} = {}^{0}_{n}J\dot{q}} $$

WHERE ${}^{0}_{n}J$ IS CALLED THE 'MANIPULATOR JACOBIAN' OR JUST 'JACOBIAN'

$$ {}^{0}_{n}J = J = [J_1\ J_2 \cdots J_n] \quad (6 \times n \text{ MATRIX}) $$

$$ \boxed{J_i = \begin{cases} \begin{bmatrix} \hat{z}_{i-1} \times (O_n - O_{i-1}) \\ \hat{z}_{i-1} \end{bmatrix} & \text{FOR REVOLUTE JOINT} \\ \begin{bmatrix} \hat{z}_{i-1} \\ 0 \end{bmatrix} & \text{FOR PRISMATIC JOINT} \end{cases}} $$

O_n : DISTANCE BETWEEN (O_0, x_0, y_0, z_0) AND (O_n, x_n, y_n, z_n)
OR ~~DISTANCE FROM~~ JUST POSITION OF O_n IN THE BASE FRAME. (

O_{i-1} : DISTANCE BETWEEN (O_0, x_0, y_0, z_0) AND $(O_{i-1}, x_{i-1}, y_{i-1}, z_{i-1})$
OR JUST THE POSITION OF O_{i-1} IN THE BASE FRAME. (

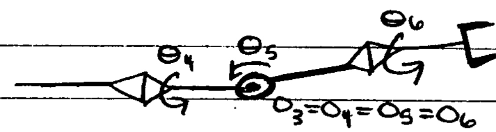
$\hat{z}_{i-1}$: COORDINATES OF $\hat{z}_{i-1}$ IN THE BASE FRAME (i.e. ${}^0_{i-1}T\,\hat{z}_0 = \hat{z}_{i-1}$

* INVERSE KINEMATICS : $\boxed{\dot{q} = J^{-1} \begin{bmatrix} {}^0_nV \\ {}^0_n\omega \end{bmatrix}}$

KINEMATIC SINGULARITIES : GIVEN BY $\boxed{\det(J) = 0}$, LEADING TO NO UNIQUE SOLUTION FOR J^{-1}.

KINEMATIC DECOUPLING (FOR SPHERICAL WRIST ROBOT MANIPULATORS)

→ WRIST SINGULARITIES



→ ARM SINGULARITIES

CONSIDER 6DOF ROBOT MANIPULATOR /W SPHERICAL WRIST

JACOBIAN: $J = \begin{bmatrix} J_{11} & \hat{z}_3 \times (O_6 - O_3) & \hat{z}_4 \times (O_6 - O_4) & \hat{z}_5 \times (O_6 - O_5) \\ J_{21} & \hat{z}_3 & \hat{z}_4 & \hat{z}_5 \end{bmatrix}$

ARM — WRIST (REVOLUTE JOINTS)

BUT $O_6 = O_5 = O_4 = O_3$ $\therefore$ $J = \begin{bmatrix} J_{11} & 0 \\ J_{21} & J_{22} \end{bmatrix}$

THEN $\det(J) = \underbrace{\det(J_{11})}_{\text{GIVES ARM SINGULARITIES}} \underbrace{\det(J_{22})}_{\text{GIVES WRIST SINGULARITIES}}$

SINGULARITIES:

ARM → $\det(J_{11}) = 0$

WRIST → $\det(J_{22}) = 0$ → $\theta_5 = 0, \pi$

FOR JACOBIAN MATRICES THAT ARE NOT SQUARE

$\boxed{\dot{q} = J^T (J J^T)^{-1} \dot{X}}$ (PROOF IN NOTES)

DEF: $J^* = J^T (J J^T)^{-1}$ (PSEUDO INVERSE JACOBIAN)

$\boxed{\dot{q} = J^* \dot{X}}$ J : $6 \times n$ JACOBIAN MATRIX

(Ch 6) DYNAMICAL MODELLING OF ROBOT MANIPULATORS

TWO COMMON METHODS: ① EULER-LAGRANGE (ENERGY BASED)
② NEWTON-EULER (FORCE/TORQUE BASED)

EULER-LAGRANGE METHOD (ENERGY BASED METHOD)

- EXPRESSIONS FOR KINETIC AND POTENTIAL ENERGIES FOR ROBOT MANIPULATORS

① KINETIC ENERGY: TWO COMPONENTS - ① TRANSLATIONAL ENERGY: CONCENTRATING MASS AT CENTER OF MASS (LINEAR MOTION)
② ROTATIONAL ENERGY: ENERGY OF THE LINK ABOUT THE CENTER OF MASS. (ROTATIONAL MOTION)

$$K_E = \frac{1}{2}\int_{LINK} v^T(x,y,z)\,v(x,y,z)\,dm \quad (v \neq f(m);\ m_i = \text{CONSTANT})$$

$$\boxed{K_{E_i} = \underbrace{\frac{1}{2} v_{c_i}^T v_{c_i} m_i}_{\text{TRASLATIONAL PART}} + \underbrace{\frac{1}{2}\omega_i^T I_i \omega_i}_{\text{ROTATIONAL PART}}} \quad \left(\text{KINETIC ENERGY OF LINK } i\right)$$

NOTE: ① $v_{c_i}^T v_{c_i} = (R\bar{v}_{c_i})^T (R\bar{v}_{c_i}) = \bar{v}_{c_i}^T R^T R \bar{v}_{c_i} = \bar{v}_{c_i}^T \bar{v}_{c_i}$

② $\omega_i^T I_i \omega$ MUST BE W.R.T. THE MOVING FRAME ATTACHED TO LINK i

m_i = MASS OF LINK i (~CONSTANT)

v_{c_i} = LINEAR VELOCITY OF THE CENTER OF MASS OF LINK i EXPRESSED IN FRAME $\emptyset$ (BASE)

ω_i = ANGULAR VELOCITY OF THE LINK i EXPRESSED W.R.T. THE FRAME ATTACHED TO LINK i

I_i = MOMENT OF INERTIA OF LINK i W.R.T. A FRAME PARALLEL TO THE LINK i FRAME WITH ORIGIN LOCATED AT THE CENTER OF MASS.

$$I_i = \begin{bmatrix} I_{xx_i} & 0 & 0 \\ 0 & I_{yy_i} & 0 \\ 0 & 0 & I_{zz_i} \end{bmatrix} = \begin{bmatrix} \int(y^2+z^2)dm & 0 & 0 \\ 0 & \int(x^2+z^2)dm & 0 \\ 0 & 0 & \int(x^2+y^2)dm \end{bmatrix}$$

$$v_{c_i} = J_{v_{c_i}}\dot{q}$$
$$\omega_i = {}^0_iR^T J_{\omega_i}\dot{q}$$

WHERE, $J = \begin{bmatrix} J_{v_{c_i}} \\ J_{\omega_i} \end{bmatrix}$ = MANIPULATOR JACOBIAN

$$\dot{q} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \vdots \\ \dot{q}_n \end{bmatrix} = \text{JOINT VELOCITIES}$$

$$\therefore \quad KE_{TOTAL} = \sum_{i=1}^{n} KE_i = \sum_{i=1}^{n} \left(\frac{1}{2} (J_{v_{c_i}} \dot{q})^T (J_{v_{c_i}} \dot{q}) m_i + \frac{1}{2} ({}^0_iR^T J_{\omega_i} \dot{q})^T I_i ({}^0_iR^T J_{\omega_i} \dot{q}) \right)$$

$$* \quad \boxed{KE_{TOTAL} = \frac{1}{2} \dot{q}^T \underbrace{\sum_{i=1}^{n} \left(J_{v_{c_i}}^T J_{v_{c_i}} m_i + J_{\omega_i}^T \, {}^0_iR \, I_i \, {}^0_iR^T J_{\omega_i} \right)}_{D(q)} \dot{q}}$$

- TOTAL KINETIC ENERGY FOR ROBOT MANIPULATOR

$$* \quad \boxed{KE_{TOTAL} = \frac{1}{2} \dot{q}^T D(q) \dot{q}}$$

WHERE $D(q)$: INERTIA MATRIX

2 POTENTIAL ENERGY:

$$* \quad \boxed{P_{E_i} = -g^T r_{c_i} m_i}$$

(POTENTIAL ENERGY OF LINK i)

g: ~~FORCE~~ ACCELERATION DUE TO GRAVITY
m_i: MASS OF LINK i
r_{c_i}: DISTANCE FROME BASE PLANE TO CENTER OF MASS OF LINK i

- TOTAL POTENTIAL ENERGY FOR ROBOT MANIPULATOR

$$* \quad \boxed{P_{E_{TOTAL}} = \sum_{i=1}^{n} P_{E_i}}$$

3 LAGRANGIAN:

TOTAL LAGRANGIAN OF ROBOT MANIP.

$$\boxed{L = K_{E_{TOTAL}} - P_{E_{TOTAL}}}$$

4 EULER-LAGRANGE EQUATIONS:

- ONE EULER-LAGRANG EQUATION FOR EACH OF THE n LINKS.

$$\boxed{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = \tau_i} \quad i = 1, 2, 3, \ldots, n$$

τ_i: GENERALIZED FORCE $\begin{cases} \tau_i = \text{FORCE IF } q_i \text{ IS PRISMATIC} \\ \tau_i = \text{TORQUE IF } q_i \text{ IS REVOLUTE} \end{cases}$

IN COMPACT MATRIX FORM, THE DYNAMICAL MODEL OF ANY nDOF ROBOT MANIPULATOR IS GIVEN BY:

$$* \quad \boxed{D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau} \qquad G(q) = \frac{\partial P_E}{\partial q_k}$$

$$D(q) = \sum_{i=1}^{n} \left(m_i J_{v_{c_i}}^T J_{v_{c_i}} + J_{\omega_i}^T \, {}^0_iR \, I_i \, {}^0_iR^T J_{\omega_i} \right), \quad D = [d_{ij}(q)]$$

$$C(q,\dot{q}) = \frac{1}{2} \sum_{i=1}^{n} \left\{ \frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k} \right\} \dot{q}_i \dot{q}_j$$

14

Newton-Euler Method (Force Based Method)

Based on the following principles:

1. Every action has reaction
2. $\dfrac{d(mv)}{dt} = \sum F \xrightarrow[\text{simple}]{\text{robot manipulators}} \boxed{m\dfrac{dv}{dt} = \sum F}$ (m = constant scalar)
3. $\dfrac{d(I_o \omega)}{dt} = \underbrace{\sum \tau_o}_{\text{about origin}} \xrightarrow{\text{simple robot manipulators}} I\dot{\omega} + \underbrace{\omega \times (I\omega)}_{\text{simple systems this term}} = \sum \tau$

   $\boxed{I\dot{\omega} = \sum \tau}$ (I = constant matrix)

   I_o: Moment of inertia of the body about an inertial frame whose origin is at the center of mass

Definitions: ~~(vectors w.r.t. frame i)~~

a_{ci} : Acceleration of the center of mass of link i wrt frame i

a_{ei} : Acceleration of the end ~~effector~~ of link i (joint i+1) wrt frame i

ω_i : Angular velocity of frame i w.r.t. frame Ø

α_i : Angular acceleration of frame i wrt frame Ø

g_i : Acceleration due to gravity (expressed in frame i)

f_i : Force exerted by link (i-1) on link i

τ_i : Torque exerted by link (i-1) on link i

${}^{i}_{i+1}R$: Rotation matrix from frame (i+1) to frame i (A_{i+1})

m_i : Mass of link i

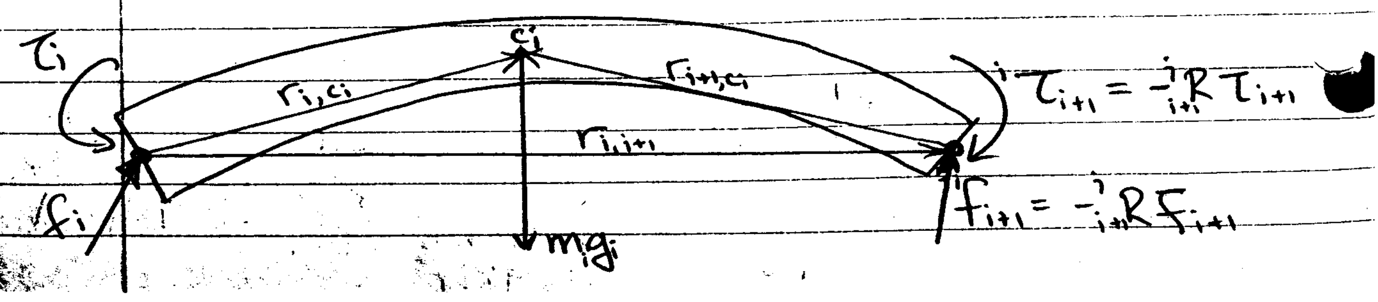
I_i : Inertia matrix of link i about a frame parallel to frame i @ center

$r_{i,ci}$: Vector from joint i to center of mass of link i

$r_{i+1,ci}$: Vector from joint (i+1) to center of mass of link i

$r_{i,i+1}$: Vector from joint i to joint (i+1)

General Free Body Diagram of Link i:



PROCEDURE:

STEP 1: FORWARD RECURSION: START WITH I.C.'s: $\boxed{\omega_0=\emptyset, \alpha_0=\emptyset, a_{c,0}=\emptyset, a_{e,0}=\emptyset}$

THEN SOLVE IN ORDER THE FOLLOWING:

1. $\boxed{\omega_i = ({}^{i-1}_{i}R)^T \omega_{i-1} + \hat{b}_i \dot{q}_i}$, WHERE $\boxed{\hat{b}_i = {}^{0}_{i}R^T \hat{z}_{i-1}}$

2. $\boxed{\alpha_i = ({}^{i-1}_{i}R)^T \alpha_{i-1} + \hat{b}_i \ddot{q}_i + \omega_i \times \hat{b}_i \dot{q}_i}$

3. $\boxed{a_{e,i} = ({}^{i-1}_{i}R)^T a_{e,i-1} + \omega_i r_{i,i+1} + \omega_i \times (\omega_i \times r_{i,i+1})}$

4. $\boxed{a_{c,i} = ({}^{i-1}_{i}R)^T a_{e,i-1} + \dot{\omega}_i \times r_{i,c_i} + \omega_i \times (\omega_i \times r_{i,c_i})}$

FOR $i = 1, 2, \ldots, n$: DETERMINE: $\omega_i, \alpha_i, a_{e,i}, a_{c,i}$

$\boxed{\text{STARTING @ } i=1}$

STEP 2: BACKWARD RECURSION: START WITH IC's: $\boxed{f_{n+1}=0, \tau_{n+1}=0}$

THEN SOLVE THE FOLLOWING:

1. $\boxed{f_i = {}^{i}_{i+1}R\, f_{i+1} + m_i a_{c_i} - m_i g_i}$

2. $\boxed{\tau_i = {}^{i}_{i+1}R\, \tau_{i+1} - f_i \times r_{i,c_i} + ({}^{i}_{i+1}R f_{i+1}) \times r_{i+1,c_i} + I\alpha_i + \omega_i \times (I\omega_i)}$

FOR $i = 1, 2, \ldots n$: DETERMINE: f_i, τ_i

$\boxed{\text{STARTING @ } i=n}$

16

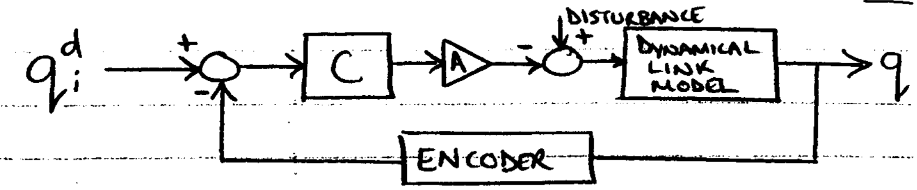
(STARTS FROM Q7) CONTROL DESIGN FOR ROBOT MANIPULATORS

① LINEAR CONTROL DESIGN

INDEPENDENT JOINT CONTROL (Ch. 7)

- EACH LINK IS CONTROLLED AS A S.I.S.O. – LINEAR SYSTEM
- ANY COUPLING EFFECTS DUE TO MOTION OF OTHER LINKS IS TREATED AS DISTU

FOR EACH LINK:



$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} J_{vc_i} \\ J_{\omega_i} \end{bmatrix} \qquad \omega_2 = \dot{q}_2 \qquad V_{c_i} = J_{vc_i}\dot{q}$$

$$V_{c_i}^T = \dot{q}^T J_{vc_i}^T$$

KINETIC ENERGY OF ROBOT MANIPULATOR:

$$KE_i = \frac{1}{2} m V_{c_i}^T V_{c_i} + \frac{1}{2}\omega^T I_i \omega = KE_{TRANS} + KE_{ROT}$$

$$KE_i = \frac{1}{2} m \dot{q}^T J_{vc_i}^T J_{vc_i} \dot{q} + \frac{1}{2}\dot{q}^T J_{\omega_i}^T \, {}^0_iR \, I_i \, {}^0_iR^T J_{\omega_i} \dot{q}$$

$$KE_i = \frac{1}{2}\dot{q}^T \left(J_{vc_i}^T J_{vc_i} + J_{\omega_i}^T \, {}^0_iR \, I_i \, {}^0_iR^T J_{\omega_i} \right) \dot{q}$$

$$\boxed{KE_{TOTAL} = \frac{1}{2}\dot{q}^T D(q) \dot{q}}$$

Where: $\boxed{D(q) = \sum_{i=1}^{n} \left(m_i J_{vc_i}^T J_{vc_i} + \underbrace{J_{\omega_i}^T \, {}^0_iR \, I_i \, {}^0_iR^T J_{\omega_i}} \right)}$

POTENTIAL ENERGY OF ROBOT MANIPULATOR:

$$PE_i = -g^T r_{c_i} m_i$$

m_i — MASS OF LINK i (CONSTANT FOR LINK i)

r_{c_i} — DISTANCE BETWEEN TWO $\perp$ TO FORCE OF GRAVITY PLANES (BASE PLANE $\longmapsto$ PLANE OF CENTER OF M.)

$$g = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$PE_{TOTAL} = -g^T \sum_{i=1}^{n} r_{c_i} m_i = \sum_{i=1}^{n} PE_i$$

LAGRANGIAN OF ROBOT MANIPULATOR

$$L = KE_{TOTAL} - PE_{TOTAL}$$

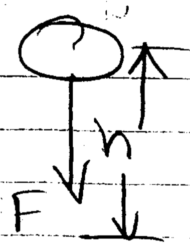
$$L = \frac{1}{2}\dot{q}^T D(q) \dot{q} + \sum_{i=1}^{n} PE_i$$

EULER-LAGRANGE EQUATIONS (FOR EACH LINK i)

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = \tau_i$$

τ_i — FORCE/TORQUE (PRISMATIC/REVOLUTE JOINT)

$V = hF \quad [N \cdot m]$



$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = \tau_i$$

$$L = K - V$$

$$KE_{TOTAL} = KE_{TRAN} + KE_{ROT}$$

$$KE_i = \frac{1}{2} m(\dot{x}_G^2 + \dot{y}_G^2) + \frac{1}{2} I\omega^2$$

$$= \frac{1}{2} m \underbrace{V_c^T V_c}_{\text{VELOCITY OF COM.}} + \frac{1}{2} \underbrace{\omega^T I \omega}_{\text{ROTATIONAL SPEED ABOUT THE CENTER OF MASS OF LINK i}}$$

CONSTANT

VARIABLE

Same irrespect to frame in which V_c is expressed

$$\omega = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

$I_{11} = \int (y^2 + z^2) \rho \, dx\, dy\, dz$

$I_{12} = -\int xy \, dm$

$I_{13} = -\int xz \, dm$

$I_{21} = -\int xy \, dm$

$I_{22} = \int (x^2 + z^2) dm$

$I_{23} = -\int (yz) dm$

$I_{31} = -\int xz \, dm$

$I_{32} = \int zy \, dm$

$I_{33} = \int (x^2 + y^2) dm$

$$I = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{bmatrix}$$

$$V_c = \begin{bmatrix} V_{cx} \\ V_{cy} \\ V_{cz} \end{bmatrix}$$

$V_{ci} = J_{vci}(q)\dot{q}$

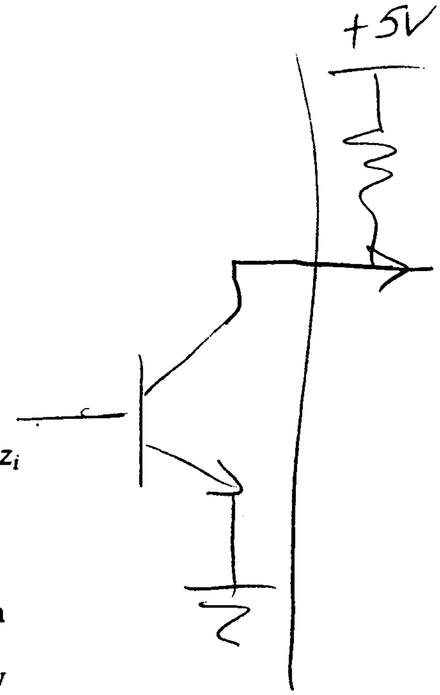
$\omega_i = {}^0_i R_i^T J_{\omega i}(q)\dot{q}$

$$I_i = \begin{bmatrix} \int (y^2+z^2) dm & 0 & 0 \\ 0 & \int (x^2+z^2) dm & 0 \\ 0 & 0 & \int (y^2+x^2) dm \end{bmatrix}$$

Hilroy

3.2.1 SUMMARY

We may summarize the above procedure based on the D–H convention in the following algorithm for deriving the forward kinematics for any manipulator.



Step 1: Locate and label the joint axes $z_0, \cdots, z_{n-1}$.

Step 2: Establish the base frame. Set the origin anywhere on the z_0-axis. The x_0 and y_0 axes are chosen conveniently to form a right-hand frame.

For $i=1, \cdots n-1$, perform Steps 3 to 5.

Step 3: Locate the origin o_i where the common normal to z_i and z_{i-1} intersects z_i. If z_i intersects z_{i-1} locate o_i at this intersection. If z_i and z_{i-1} are parallel, locate o_i at joint i.

Step 4: Establish x_i along the common normal between z_{i-1} and z_i through o_i, or in the direction normal to the z_{i-1}-z_i plane if z_{i-1} and z_i intersect.

Step 5: Establish y_i to complete a right-hand frame.

Step 6: Establish the end-effector frame $o_n x_n y_n z_n$. Assuming the n-th joint is revolute, set $\mathbf{k}_n = \mathbf{a}$ along the direction z_{n-1}. Establish the origin o_n conveniently along z_n, preferably at the center of the gripper or at the tip of any tool that the manipulator may be carrying. Set $\mathbf{j}_n = \mathbf{s}$ in the direction of the gripper closure and set $\mathbf{i}_n = \mathbf{n}$ as $\mathbf{s}\times\mathbf{a}$. If the tool is not a simple gripper set x_n and y_n conveniently to form a right-hand frame.

Step 7: Create a table of link paramoters $a_i, d_i, \alpha_i, \theta_i$.

a_i = distance along x_i from o_i to the intersection of the x_i and z_{i-1} axes.

d_i = distance along z_{i-1} from o_{i-1} to the intersection of the x_i and z_{i-1} axes. d_i is variable if joint i is prismatic.

α_i = the angle between z_{i-1} and z_i measured about x_i (See Figure 3-3).

θ_i = the angle between x_{i-1} and x_i measured about z_{i-1} (See Figure 3-3). θ_i is variable is joint i is revolute.

Step 8: Form the homogeneous transformation matrices A_i by substituting the above parameters into (3.2.1).

Step 9: Form $T_0^n = A_1 \cdots A_n$. This then gives the position and orientation of the tool frame expressed in base coordinates.

$$A_i = \begin{bmatrix} C_{\theta_i} & -S_{\theta_i} C_{\alpha_i} & S_{\theta_i} S_{\alpha_i} & a_i C_{\theta_i} \\ S_{\theta_i} & C_{\theta_i} C_{\alpha_i} & -C_{\theta_i} S_{\alpha_i} & a_i S_{\theta_i} \\ 0 & S_{\alpha_i} & C_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$